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FLOW BOILING: PREDICTION OF BUBBLE DEPARTURE

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INTRODUCTION

AN EARLIER paper [1] investigated the bubble diameter on detachment of gas bubbles adhering to the wall of a pipe with liquid flowing past. Theoretical expressions for the drag, surface tension and buoyancy forces acting on the bubble were checked and then corrected against the experimentally measured bubble departure diameters. Apart from the basic interest of these novel measurements there was a practical requirement in a number of systems to be able to predict gas bubble behaviour. In particular gas bubbles adhering to heat transfer surfaces can seriously impede the flow of heat [2], and gas bubbles originating elsewhere in the system can form the nuclei required to initiate boiling [3, 4].

At an early stage though it was realized that there was another potential application of the equations that had been obtained for the various forces: and that was to predict the size of vapour bubbles departing from the surface in forced convective boiling. In this field also comparatively little experimental work has been done [5, 6], in contrast to the much more extensive literature on bubble size in pool boiling. A number of workers have been interested in a closely related phenomenon, that of predicting the void fraction in subcooled boiling [7–9]. These workers have placed the emphasis on predicting the superheat at the point of bubble departure and the void fraction rather than on the bubble departure size as such.

In addition to the basic drag, surface tension and buoyancy forces, it has been suggested that many other forces may play an important role in vapour bubble formation [10]. These forces include inertial forces (due to the rapidity of the vapour growth), lift forces (due to circulation around the bubble), an excess pressure force (due to the difference in pressure between the inside and the outside of the bubble) and a capillarity force (due to circulation of the surface of the bubble under the surface tension gradient). Also many workers have reported that at a late stage in the formation of the vapour bubble, just before it detaches from the surface, it is connected to the surface by a neck [5, 6].

Since the present author's earlier work on gas bubbles provided little or no information on some of these extra forces and there had been no sign of a neck attaching the bubble to the surface, there was some initial hesitation in applying the same ideas to forced convective boiling.

However, the logic of the previous analysis, which is that it is a force balance parallel to the wall which is critical in determining bubble departure, means that many of the new extra forces are irrelevant, in fact all those that act perpendicular to the wall. Also there is recent evidence that the neck attaching the bubble to the wall may be an illusion, resulting from the temperature gradient in the liquid near the wall producing a variation in the refractive index [11]. Consequently, with the discovery of some experimental results for flow boiling that were almost ideally suited for a check of the analysis [5], it was decided to use the computer program previously developed for gas bubbles. No changes at all were made to the program itself, though obviously new values of physical properties, duct equivalent diameter and flow orientation were used. The results, as will be seen, are very encouraging.

Considerable progress still needs to be made in the theoretical understanding of forced convective boiling. It is hoped that an ability to predict bubble sizes will help towards an eventual understanding.

There is insufficient space in this note to reproduce the theory of the method used to predict the bubble departure size in detail. The main equations required are available in the earlier paper [1].

COMPARISON WITH EXPERIMENTAL RESULTS

First the experimental results of Koumoutsos *et al.* [5] are considered. These results were obtained in forced convection boiling of water in a horizontal duct at around atmospheric pressure. The duct dimensions were 26 mm wide by 15 mm high. Single vapour bubbles were formed on the floor of the duct, and recorded using high-speed photography. To correspond with the theoretical equations given earlier [1] it was important that the vapour bubble be attached to a smooth surface. In this experiment, it seems that this condition was satisfied. Although a small nucleating cavity was made at the point where it was desired that the bubble should form, it was evident that the vapour region spreads beyond this cavity before it detaches into the flow. The rest of the surface was polished to a mirror finish. Confirmation of the small size of the nucleating cavity comes from other information given about the nucleating behaviour (the cavity size was unfortunately not quoted in the paper). The superheat for most of the measurements was 15°C, which can be interpreted as a cavity radius of 1.6 μm. Also problems were experienced with nucleation from natural cavities which obscured the view, until measures were taken to cool the duct walls away from the desired nucleation site.

The experimental points are shown in Fig. 1. In constructing Fig. 1 the tabulated data given in ref. [5] was used rather than the graph. For some reason a couple of data points at low velocity, present in the table, were omitted from the graph. For the theoretical comparison the equations given in ref. [1] were used. The only difficulty is that these equations require values of the advancing and receding contact angles, which were not recorded. In the earlier study [1], where these angles were recorded, it was found that the average difference between the advancing and receding contact angles was 20°. Also the difference did not seem to be dependent on the actual value of the average or equilibrium contact angle. The equilibrium contact angles varied over the range 22°–90°. Throughout this range then it seems reasonable to take $\theta_r = \theta_0 - 10$ and $\theta_a = \theta_0 + 10$, and this is what was done for the present calculation. The measured average contact angles presented by Koumoutsos *et al.* show a small variation from one measurement to the next. The theoretical prediction of Fig. 1 is based on an average equilibrium contact angle of 37.6°.

An interesting feature of the theoretical calculation is that, as a result of the geometry chosen by Koumoutsos *et al.*, the component of the buoyancy force parallel to the surface is zero, i.e. the buoyancy force is effectively ignored. At very low velocities this approach is bound to prove inadequate, and for this reason the theoretical prediction is not extended to velocities below 0.15 m s^{-1}. At lower velocities the full, vertical, buoyancy force becomes larger than the horizontal drag force.

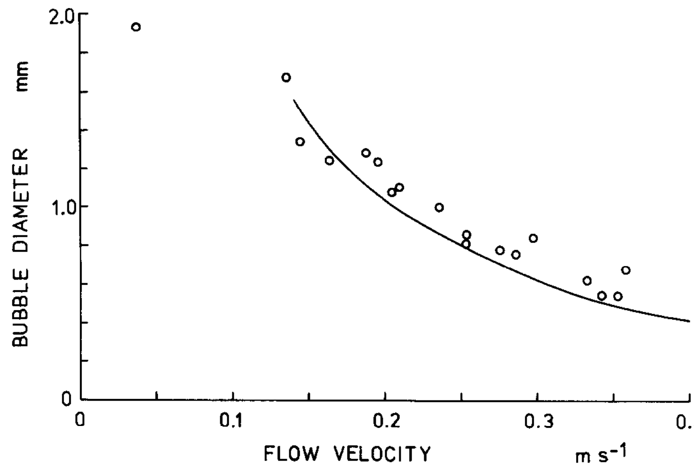

FIG. 1. Bubble departure diameter for boiling water at a pressure of 1 atm. The theory (solid line) is compared with experimental results from ref. [5].

The data presented by Koumoutsos *et al.* seemed to be the only data that completely corresponded to the situation for which the original computer program was written. The experimental arrangement used by Chullabodhi [6] though was very nearly suitable, and it was decided that it would be of interest to make the comparison even though the model was not completely appropriate. The problem lies in the fact that Chullabodhi studied vapour bubble formation at quite large artificial drilled cylindrical cavities, from 60 to 1000 μm in diameter. There was evidence that the vapour bubbles were attached to the lip of these cavities and did not spread out over the surface. The theory of this paper would, of course, be most likely to apply to the smallest cavity, so it was decided to confine the comparison to the experimental results for the smallest, 60 μm cavity size. Apart from this difficulty, there appeared to be significant advantages in testing the theory against Chullabodhi's data. Results are presented for three different liquids. Also considerable information is available on the contact angles, though the interpretation of this information is complicated by the facts that at times the wetting line, i.e. the line of three phase contact, was probably attached to the lip of the cavity. Bearing this reservation in mind, and the estimate in ref. [6] that there could have been an error of up to 5° in the contact angle measurements, then the results are entirely consistent with the earlier evidence for a 20° difference between the advancing and receding contact angles.

In other respects the experimental arrangement used by Chullabodhi was similar to that of Koumoutsos *et al.*

A comparison has been performed for each of the three liquids, but only the results for n-heptane are shown here (Fig. 2). The other two liquids show a very similar degree of agreement between theory and experiment and very similar trends with change of velocity. The experimental points in Fig. 2 are presented with error bands. Given the doubts expressed earlier about the applicability of the theory, the agreement is encouraging.

As in the comparison with Koumoutsos *et al.*'s results there is evidence at low velocities, below about 0.15 m s^{-1}, that the theory is inadequate. At higher velocities the measured bubble departure diameters tend to be higher than the prediction, though in virtually all cases a doubling of the experimental error band would bring the two into agreement. A more reasonable explanation of the discrepancy though is that experimental results obtained on a smoother surface would show smaller diameters.

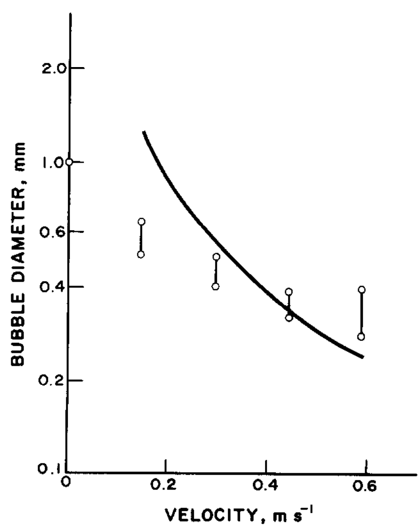

FIG. 2. Bubble departure diameter for boiling n-heptane. The theory (solid line) is compared with data from ref. [6] for bubbles leaving a 60 μm cavity.

DISCUSSION

Both graphs suggest that at very low flow velocities a different mechanism of bubble detachment becomes relevant. It would of course have been possible to modify the theory to allow for this alternative mechanism, with the diameter at detachment being chosen according to whichever mechanism first gave rise to detachment as the vapour region grew, and it is likely that this extended theory would have given very good agreement with the experimental data. However, it was felt important to test the theory in a completely unchanged form so as to give confidence that it could be applied to vapour bubbles and that there were no new forces that had a significant role to play. The results do confirm the applicability of the theory previously used for gas bubbles.

CONCLUSION

A theory of bubble detachment into flowing liquids previously only applied to slow growing gas bubbles may also be applied to vapour bubbles in boiling.

Acknowledgement—I would like to thank Dr D. James of UMIST for drawing my attention to ref. [6] and making available a copy of it.

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UNSTEADY NONSIMILAR NATURAL CONVECTION OVER A VERTICAL FLAT PLATE IN A THERMALLY STRATIFIED FLUID

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NOMENCLATURE

a	ambient temperature gradient, dT_∞/dx
C_f	local skin-friction coefficient
F, F_w	dimensionless stream function and mass-transfer parameter, respectively
F''_w, G'_w	surface skin-friction and heat-transfer parameters, respectively
g, G	gravitational acceleration and dimensionless temperature, respectively
Gr_x, k	local Grashof number and thermal conductivity, respectively
Nu, Pr	local Nusselt number and Prandtl number, respectively
q	local heat-transfer rate per unit area
t, t^*	dimensional and dimensionless times, respectively
$T, T_\infty, T_{\infty 0}$	temperature, ambient temperature, and ambient temperature at $x = 0$, respectively
T_w, T_{w0}	wall temperature and its value at $t = 0$, respectively
x, y	distances along and perpendicular to the surface, respectively.

Greek symbols

β	bulk coefficient of thermal expansion
ε	constant
η, ξ	transformed coordinates
ν, ρ	kinematic viscosity and density, respectively
τ, ϕ, ψ	shear stress at the surface, function of t, and dimensional stream function, respectively.

Superscript

$'$	differentiation with respect to η.

Subscripts

w, ξ	condition at the surface and derivative with respect to ξ, respectively.

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INTRODUCTION

THE AMBIENT fluid in many natural or mixed convection flows both in technology and in nature is stably stratified. Thermal stratification is commonly encountered in the atmosphere and in lakes and cooling ponds. Also heat rejection from power plants and other industrial systems often involves both natural and mixed convection flows in stratified media. The steady free convection flow over a vertical flat plate in a stably stratified medium has been studied by several authors [1–4]. However, the analogous unsteady problem has not been studied so far.

The aim of this note is to study the unsteady incompressible laminar free convection boundary layer for nonsimilar flow over a vertical flat plate in a stratified medium when the wall temperature varies with time. The partial differential equations, with three independent variables governing the flow, have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique [5, 6]. The results have been compared with those available in the literature.

GOVERNING EQUATIONS

We consider a vertical plate immersed in a stable thermally stratified fluid. The wall temperature is assumed to vary continuously with time and the ambient temperature with distance from the leading edge. The boundary layer equations for the unsteady laminar incompressible natural convection flow on a vertical flat plate with mass transfer using the Boussinesq approximations for the density variation and